Counting Statistics

Radioactive decays follow some curious rules that are a consequence of quantum mechanics. Regardless of when a particular nucleus was created, all nuclei of the same species (Cobalt-60 in this experiment) have exactly the same probability of decay. We might expect that the longer a nucleus has been around, the more likely it is to decay, but that is not what is observed. Even though the *probability* that a given nucleus will decay is fixed, there is no way to predict *when* it will decay. In this sense the decay process is completely random. Despite this randomness, a collection of many identical and independent nuclei will exhibit certain predictable behaviors, such as a consistent average decay rate when measured over a long time.

There are still variations in the average count rate when measured over a shorter time, however. Suppose we collect data on the number of decays during a five-second interval. We count decays for five seconds, and then another five, and so forth. If the average number of counts during each interval is n, then we will find that the standard deviation of the collection of measurements is on average $n^{1/2}$. The standard deviation is a measure of how far away, on average, a measurement is from the mean value. A histogram of the measurements of the number of decays detected each interval will show the characteristic distribution known as the *Poisson distribution*.

When the average number of decays each interval is small, such as one or two, then the Poisson distribution is not symmetric. An asymmetric distribution means that the most common value is different from the average value. If the average number of decays in each time interval is larger, such as more than twenty, the shape of the Poisson distribution approaches the shape of the Normal, or Gaussian, distribution. The Normal distribution is sometimes called the *bell-shaped curve*, although there are other distributions that also look like a bell! The Normal distribution is symmetric, with the average value being identical to the most common value.

In this experiment you will collect data from a source that exhibits an essentially constant decay rate. Because the lifetime of the source is so long, the average decay rate will not change during your experiment. The interval-to-interval count rate will vary, however, but in a way consistent with the Poisson distribution.

OBJECTIVES

- Use a radiation counter to determine the distribution of count rates from a nearly constant-rate source.
- Compare the distribution of experimental nuclear counting data to the Poisson distribution.
- Observe the gradual transition of count distribution from Poisson statistics to Gaussian statistics as the average count rate increases.

MATERIALS

computer	Radiation Monitor
Vernier computer interface	Cobalt-60 1 μC disc gamma source
Logger *Pro*	

PRELIMINARY QUESTIONS

1. Switch your radiation monitor to AUDIO mode, and place it about ten centimeters from your Co-60 source. When the monitor detects a by-product of a decaying Co-60 nucleus (a gamma ray, in this case) it emits a beep and the red LED flashes. Is there a uniform time between beeps? Or does the time vary? From listening to the sequence of beeps, can you predict when the next beep will occur?

2. Now move the source closer to the monitor. Did the average rate of beeps appear to change? If so, how did it change? Is there any more or less uniformity to the time interval between beeps compared to the slower rate?

PROCEDURE

1. Connect the radiation monitor to DIG/SONIC 1 of the interface. Switch the Monitor to the ON (no audio) position.

2. Prepare the computer for data collection by opening the file "04 Statistics" from the *Nuclear Radiation w Vernir* folder of Logger *Pro*. Two graphs are displayed: a histogram of the count rate and the counts *vs.* time.

3. Place the radiation monitor on top of or adjacent to the Co-60 source so that the rate of flashing of the red LED is maximized.

4. Click ▶Collect to begin collecting data. Logger *Pro* will begin counting the number of gamma photons that strike the detector during each one-second count interval. Data collection will continue for just 30 seconds. Do not move the detector or the source for the remainder of data collection.

5. After data collection is complete, the ▶Collect button will reappear.

6. To study the variation in count rate distributions, you will need to change the length of one time interval so that the average number of counts is first small (about 1) and then larger (30 or so). The count rate from your particular source depends on its age and initial activity, so you will need to first determine the average count rate from your sample. To do this, click on the graph of counts/interval *vs.* time. Then click on the Statistics button, to see the average count rate. Enter this value in your data table. Now determine the necessary interval lengths to achieve an average of 1 count per interval and an average of 30 counts per interval. Round these values up to the next 0.05 second. For example, let's say your average count rate was 4.67 counts per one-second interval. To get about one count per interval with the same source, you would use an interval of 4.67^{-1} or 0.21 seconds, rounded to 0.25 seconds. For 30 counts, multiply this by 30, getting 6.45 s after rounding up. Enter these values in your data table.

7. Set the counting interval to the value needed to obtain an average count of approximately one. To do this, select Data Collection from the Experiment menu. Change the number of seconds/sample to the low count rate value in your data table. Take care not to use the samples/second field. Then, change the Experiment Length field so that 200 samples will be collected (*i.e.*, enter a value 200 times the seconds/sample time). Click Done .

8. Click the ▶Collect button to begin counting. Observe the histogram as data are collected. Is there a regular pattern as to the next count rate that appears? Do the values appear to be clustered around a most-common value? When data collection is complete, the ▶Collect button will reappear.

9. After data collection is complete, click the Counts *vs.* Time graph to make it active, and then click on the statistics button to calculate statistics for the data. Record the average and standard deviation in your data table. Rescale your graph if needed.

10. Print your screen by selecting Print from the File menu.

11. Set the counting interval to the value needed to obtain an average count of approximately thirty. To do this, select Data Collection from the Experiment menu. Change the number of seconds/sample to the high count rate value in your data table. Then, change the Experiment Length field so that 200 samples will be collected (*i.e.*, enter a value 200 times the seconds/sample). Click $\boxed{\text{Done}}$.

12. Click the $\boxed{\blacktriangleright \text{Collect}}$ button to begin counting. Observe the histogram as data are collected. Is there a regular pattern as to the next count rate that appears? Do the values appear to be clustered around a most-common value? When data collection is complete, the $\boxed{\blacktriangleright \text{Collect}}$ button will reappear. Rescale your graph as needed.

13. Click the counts/interval *vs.* time graph to make it active, and then click on the statistics button to calculate statistics for the data. Record the average and standard deviation in your data table.

14. Print your screen by selecting Print from the File menu.

15. The standard deviation is a measure of how far away, on average, a typical measurement (of counts during each interval) is from the average of all the measurements. The interval defined by (average ± one standard deviation) contains most of the measurements. From your average and standard deviation values, determine this interval, rounded to the nearest integer. From the Histogram data table window, determine the fraction of the measurements that fall within the interval.

DATA TABLE

Average count rate (1 s interval)	

	Low count rate (~1/interval)	High count rate (~30/interval)
Interval length (s)		
Average rate (counts/interval)		
Square root (average rate)		
Standard deviation (counts/int)		
Fraction within ± std dev		

ANALYSIS

1. Is your first histogram (with the low average count rate) symmetric? How can you tell? Was that shape consistent with the Normal distribution?

2. Is your second histogram (with the high average count rate) symmetric? How can you tell? Is the symmetry of your data distribution consistent with the Normal distribution?

3. Calculate the square root of the average count rate for your low- and high-count-rate trials. The square root of the number of counts measured in one interval is an estimate of the standard deviation of a set of measurements, when those measurements follow the Poisson distribution. How does the square-root estimate compare to the actual standard deviation of your set of measurements?

4. Use the comparison in the previous question to answer this question: An experiment yields 900 counts in one interval. Predict the standard deviation of a set of 200 additional measurements made under the same conditions.

5. For your high-count-rate data, is the fraction of the measurements that fall within the interval close to two-thirds? The Normal distribution is symmetric and has two-thirds of its values within one standard deviation of the average. Is the distribution of your data consistent with the Normal distribution?

EXTENSIONS

1. Consult a statistics or nuclear physics reference book to learn the mathematical form of the Poisson distribution. Plot a Poisson distribution with the same average and standard deviation as your low-count-rate data on the same graph with those data.

2. Consult a statistics or nuclear physics reference book to learn the mathematical form of the Normal distribution. Plot a Normal distribution with the same average and standard deviation as your high-count-rate data on the same graph with those data.

3. Determine the fraction of your measurements falling with two standard deviations of the average for the high-count-rate measurements. The Normal distribution includes 90% of the measurements within two standard deviations of the average.

4. Determine the fraction of your measurements falling with three standard deviations of the average for the high-count-rate measurements. The Normal distribution includes 99% of the measurements within two standard deviations of the average.

5. Collect additional data at the high count rate. Use intervals with 500 to 1000 counts. Is the histogram different in shape from your earlier data?